St Catherine's School

Year: 12
Subject: 3 Unit Mathematics
Time Allowed: 2 hours (plus 5 mins reading time)
Date: August 2000

Exam number:______________

Directions to candidates:

- All questions are to be attempted.
- All questions are of equal value.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Each question attempted should be started on a **new** page.
- Approved calculators and geometrical instruments are required.
- Attach the question paper to the front of **Section A.**
- Write a cover page for **Section B and C** and include your number.
- Hand in your work in 3 **bundles:**
  - Section A Questions 1, 2 and 3.
  - Section B Questions. 4 and 5
  - Section C Questions. 6 and 7.

TEACHER'S USE ONLY Total Marks
A
B
C
TOTAL

Section A

Question 1

a) Differentiate $e^{2x}\sin x$

b) Find the acute angle between the lines $2x + y = 4$ and $x - y = 2$

c) A committee of 3 men and 4 women is to be formed from a group of 8 men and 6 women. Write an expression for the number of ways this can be done.

d) Evaluate $\int_0^2 \frac{dx}{4+x^2}$

e) Using the substitution $u = 2x+1$ or otherwise, find $\int_0^1 \frac{4x}{2x+1}\,dx$

Question 2

a) A particle is moving in simple harmonic motion. It's displacement, x, at time, t, is given by $x = 3\sin(4t + \frac{\pi}{4})$.

i) find the period and amplitude of the motion

ii) find the velocity of the particle when t = 0.

iii) find the maximum acceleration of the particle.

iv) find the speed of the particle when $x = 2$

b) The polynomial $P(x) = x^3 + bx^2 + cx + d$ has roots at 0, 3 and -3.

i) find b, c and d

ii) without using calculus, sketch the graph of $y = P(x)$

iii) Hence or otherwise solve the inequality $\frac{x^2 - 9}{x} \geq 0$.

Question 3

a) i) Find $\frac{dy}{dx}$ if $y = \tan^{-1}(\sin x)$ 2

ii) Evaluate $\int_0^1 \frac{dx}{\sqrt{2-x^2}}$ 2

b) A cup of hot coffee at temperature T degrees Celsius loses heat when placed in a cooler environment. It cools according to the law $\frac{dT}{dt} = k(T - T_0)$ where time, t is the time elapsed in minutes and T_0 is the temperature of the environment in degrees Celsius.

i) A cup of coffee at 100 ^0C is placed in an environment at -20 ^0C for 3 minutes and then cools to 70 ^0C. Find k. 2

ii) The same cup of coffee at 70 ^0C is then placed in an environment at 20 ^0C assuming k stays the same, find the temperature of the coffee after a further 15 minutes. 3

c) The function *h(x)* is given by $h(x) = \sin^{-1}x + \cos^{-1}x$ for $-1 \le x \le 1$.

i) show that $h'(x) = 0$ 1

ii) sketch the graph of $y = h(x)$ 2

SECTION B (Start a new page)

Question 4

a) A spherical balloon is expanding so that its volume is increasing at the constant rate of $10\,mm^3$ per second. What is the rate of increase of the radius when the surface area is $500\,mm^2$.
($V = \frac{4}{3}\pi r^3 \quad SA = 4\pi r^2$) 4

b) Find the constant term in the expansion of $(3x^2 - \frac{1}{2x})^9$. 4

c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The equation of chord PQ is given by $y - ap^2 = \frac{p+q}{2}(x - 2ap)$.

i) If PQ is a focal chord show that $pq = -1$ 1

ii) Find M, the midpoint of PQ. 1

iii) Find the equation of the locus of M 2

Question 5

A dangerous fire is burning in a low open tank on horizontal ground. Fire fighters are forced to stay 60m away from the fire. They are using a pump which is on the ground and can eject water at *30m/s* at any angle to the horizontal, α.
(Assume that g = *10m/s/s* and that all frictional forces, including air resistance, can be neglected.)

a) Show that the expression for the vertical motion is $y = -5t^2 + 30t\sin\alpha$ $1\frac{1}{2}$

b) Show that the expression for horizontal motion is $x = 30t\cos\alpha$ $1\frac{1}{2}$

c) Show that the range of the projectile is given by $x = 90\sin 2\alpha$ 2

d) Find the maximum horizontal distance the pump can reach. 1

e) Find the two angles of projection needed for the pumped water to reach the fire. 3

f) Another other pump is on a vertical stand 5m high and can eject water at *40m/s* but only horizontally. Can this pump reach the fire? Justify your answer.
(You may use the formulas for the horizontal distance; $x = Vt\cos\alpha$ and vertical distance $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$, where V is the initial velocity and α is the angle of projection and g=10m/s/s). 3

SECTION C (Start a new page)

Question 6

a) i) Show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 x\, dx = \frac{\pi}{8} - \frac{1}{4}$ 2

ii) The function $g(x)$ is given by $g(x) = 2 + \cos x$. The graph $y = g(x)$ for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$ is rotated about the x axis. Find the volume of the solid generated. (You may use the result of a(i)). Give your answer in exact form. 3

b) The velocity of a point moving along the x axis is given by $v^2 = 16x - 4x^2 + 20$.

i) Show that $\ddot{x} = -4(x-2)$ 2

ii) State the centre of motion 1

iii) What is the amplitude of the motion 2

iv) What is the period of the motion 1

v) Find the maximum speed of the particle 1

Question 7

a) If $(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$ show that $\sum_{r=1}^{n} r \binom{n}{r} = n\, 2^{n-1}$ 3

b) Consider the function $f(x) = (x-2)^2 + 1$

i) Sketch the parabola $y = f(x)$, showing clearly any intercepts with the axes, and the coordinates of its vertex. Use the same scale on both axes. 1.5

ii) What is the largest domain containing the value $x = 3$, for which the function has an inverse function $f^{-1}(x)$? 1

iii) Sketch the function $y = f^{-1}(x)$ on the same set of axes as your graph in part(i). Label the two graphs clearly. 1.5

(iv) What is the domain of the inverse function? 1

(v) Let a be a real number not in the domain found in part (ii). Find $f^{-1}[f(a)]$.2

(vi) Find the x coordinate of any points of intersection of the two curves $y = f(x)$ and $y = f^{-1}(x)$. 2

END OF EXAMINATION

Table of Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \ne -1; \quad x \ne 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \ne 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \ne 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \ne 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \ne 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \ne 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \ne 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x + \sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x + \sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Year 12 ... [cut off]

[Sec]tion A

[Questi]on 1

$\frac{d}{dx}\, e^{2x} \cdot \sin x =$

$e^{2x} \cdot \cos x + \sin x \cdot 2e^{2x}$

$e^{2x}(\cos x + 2\sin x)$ (2)

$2x + y = 4 \rightarrow y = 4 - 2x$, $m_1 = -2$

$x - y = 2 \rightarrow y = x - 2$, $m_2 = 1$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$= \left| \frac{-2-1}{1+(-2)(1)} \right|$

$= 3$

$\theta = 72^\circ$ (nearest degree) (2)

Choose 3 from 8 and 4 from 6

$\therefore$ No of ways $= {}^8C_3 \times {}^6C_4$ (1)

$= 56 \times 15$

$= 840$

$\int_0^2 \frac{dx}{4+x^2} = \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2$

$= \frac{1}{2}\tan^{-1}\left(\frac{2}{2}\right) - \frac{1}{2}\tan^{-1}\left(\frac{0}{2}\right)$

$= \frac{1}{2} \times \frac{\pi}{4} - \frac{1}{2} \times 0$

$= \frac{\pi}{8}$

(3)

e) $\int_0^1 \frac{4x}{2x+1}\, dx$

$u = 2x+1$ also $2x = u - 1$

$\frac{du}{dx} = 2$

$du = 2 \cdot dx$

$x = 1 \quad u = 3$

$x = 0 \quad u = 1$

$= \int \frac{4x}{2x+1}\, dx$

$= \int_1^3 \frac{u-1}{u}\, du$

$= \int_1^3 1 - \frac{1}{u}\, du$

$= \left[u - \log_e u\right]_1^3$

$= (3 - \log_e 3) - (1 - \log_e 1)$

$= 2 - \log_e 3$

(4)

Question 2

a) $x = 3\sin\left(4t + \frac{\pi}{4}\right)$

i) amplitude $= 3$ ($\frac{1}{2}$)

period $= \frac{2\pi}{n}$

$= \frac{2\pi}{4}$

$= \frac{\pi}{2}$ ($\frac{1}{2}$)

(ii) $v = \frac{dx}{dt}$

$= 3\cos\left(4t + \frac{\pi}{4}\right) \cdot 4$

$= 12\cos\left(4t + \frac{\pi}{4}\right)$ (1)

when $t = 0$

$v = 12\cos\left(\frac{\pi}{4}\right)$

$= 12 \times \frac{1}{\sqrt{2}}$

$= 12 \times \frac{\sqrt{2}}{2}$

$= 6\sqrt{2}$ (1)

(iii) max acc occurs when $v = 0$

$v = 12\cos\left(4t + \frac{\pi}{4}\right)$

$0 = 12\cos\left(4t + \frac{\pi}{4}\right)$

$4t + \frac{\pi}{4} = \frac{\pi}{2}$

$4t = \frac{\pi}{4}$

$t = \frac{\pi}{16}$ (1)

$a = \frac{dv}{dt}$

$= -48\sin\left(4t + \frac{\pi}{4}\right)$

at $t = \frac{\pi}{16}$

$a = -48 \sin\left(4\left(\frac{\pi}{16}\right) + \frac{\pi}{4}\right)$

$= -48\sin\left(\frac{\pi}{2}\right)$

$= -48$

(1)

$\therefore$ max acceleration is -48 m/[s²]

iv) $v^2 = n^2(a^2 - x^2)$

$n = 4$ ✓

$a = 3$ ✓

$x = 2$

when $x = 2$

$v = \pm\sqrt{80}$ ✓✓

$\therefore$ speed is $\sqrt{80}$ m/s or $4\sqrt{5}$ [cut off]

$x^3+bx^2+cx+d = x(x+3)(x-3)$

$P(0)=0 \qquad 0=d$

$P(3)=0 \qquad 0=27+9b+3c$ ①

$P(-3)=0 \qquad 0=-27+9b-3c$ ②

① + ②

$0=18b$

$b=0$ ✓

$\therefore 0=27+3c$

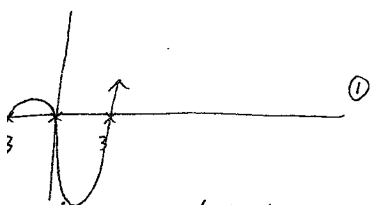
$27=-3c$

$c=-9$ ✓

$b=0, \quad c=-9, \quad d=0$ ②

ii) $y=P(x)$

$= x^3-9x$



①

at $x=1 \quad y=-8$

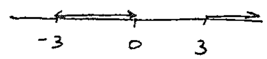
x

$\frac{x^2-9}{x}=0 \qquad x\neq 0$

$x^2-9=0$

$(x-3)(x+3)=0$

$x=3,-3$ are critical pts



at $x=1 \quad \frac{x^2-9}{x} = -8$

etc

$-3\leq x<0$ or $x\geq 3$ ②

or

$x(x^2-9) \qquad Q6\geq 0$

let $u=\sin x \qquad y=\tan^{-1}u$

$\frac{du}{dx}=\cos x \qquad dy/du = \frac{1}{1+u^2}$

$\frac{dy}{dx} = \cos x \times \frac{1}{1+\sin^2 x}$

$=\frac{\cos x}{1+\sin^2 x}$ ②

ii) $\int_0^1 \frac{dx}{\sqrt{2-x^2}} = \left[\sin^{-1}\frac{x}{\sqrt2}\right]_0^1$

$=\sin^{-1}\left(\frac{1}{\sqrt2}\right)-\sin^{-1}\frac{0}{\sqrt2}$

$=\frac{\pi}{4}$ ②

3b) $\frac{dT}{dt}=k(T-T_0) \quad \therefore T=T_0+Ae^{kt}$

i) $T_0=-20$

At $t=0, \quad T=100$

$100=-20+A$

$A=120$

at $t=3 \quad T=70$

$70=-20+120e^{3k}$

$e^{3k}=\frac{9}{12}$

$k=\frac13\ln\frac34$

$\doteqdot -0.095894...$ ②

ii) let $t=0$ when ap placed in environment $20°C$

$T=20+Be^{kt}$

at $t=0 \quad T=70°$

$70=20+Be^0$

$B=50$

$T=20+50e^{kt}$

where $k\doteqdot 0.0954894..$

at $t=15 \quad T=20+50e^{15k}$

$\doteqdot 31.86°$ ③

3c)

$h(x) = \sin^{-1}x + \cos^{-1}x \quad 0 \le x \le 1$

$h'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1+x^2}}$

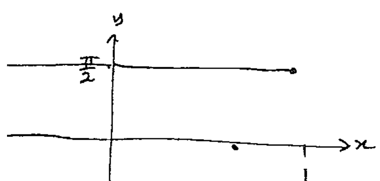
$= 0$ (1)

∴ The function $h(x)$ has a gradient of zero

∴ The fn is a straight line of the form $y = a$ where a is a constant.

when $x = 0$ $h(x) = \frac{\pi}{2}$

$\therefore h(x) = \frac{\pi}{2}$ for $0 \le x \le$



(2)

Question 4

$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi R^3\right)$ ✓✓

$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \times \frac{dR}{dt}$ ✓✓

$= 4\pi R^2 \frac{dR}{dt}$ ✓

hence $\frac{dV}{dt} = 10$ and $4\pi R^2 = 500$

$\frac{dR}{dt} = \frac{10}{500} = \frac{1}{50}$ ✓✓

SECTION B

4) b) The constant term of $\left(3x^2 - \frac{1}{2x}\right)^9$

$U_{r+1} = {}^9C_r (3x^2)^{9-r} \left(-\frac{1}{2x}\right)^r$ ✓✓

$= A (x^2)^{9-r} (x^{-1})^r$

$= A x^{18-2r} . x^{-r}$

$= A x^{18-3r}$ ✓✓

where A is the num. coeff of U_{r+1}

now $18 - 3r = 0 \therefore r = 6$ ✓✓

$\therefore u_7 = {}^9C_6 (3x^2)^{9-6} \left(-\frac{1}{2x}\right)^6$

$= \frac{9!}{6!\,3!} (3x^2)^3 \left(-\frac{1}{2x}\right)^6$

$= \frac{567}{16}$ (4)

4b) Chord PQ $y - ap^2 = \frac{p+q}{2}(x - 2ap)$

will be satisfied by $(0, a)$

$a - ap^2 = \frac{p+q}{2}(0 - 2ap)$

$a - ap^2 = (p+q)(-ap)$

$1 - p^2 = -p^2 - pq$

$1 = -pq$

$\therefore pq = -1$ (1)

ii) Midpt $PQ = \left(\frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2}\right)$

$= \left(a(p+q), \frac{a}{2}(p^2+q^2)\right)$ (1)

4b)(iii) $x = a(p+q)$

$p + q = \frac{x}{a}$

$pq = -1$

$y = \frac{a}{2}(p^2 + q^2)$

$\frac{2y}{a} = (p+q)^2 - 2pq$

$\frac{2y}{a} = \left(\frac{x}{a}\right)^2 - 2$

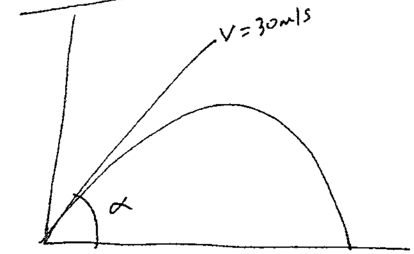
$2y = \frac{x^2}{a} - 2a$

$2ay = x^2 - 2$

$x^2 = 2ay + 2$

$x^2 = 2(ay + 1)$

Question 5



a) $\ddot{x} = 0$

$\dot{x} = t + c_1$ but when $t=0$ $\dot{x} = V\cos\alpha$

$\therefore c_1 = V\cos\alpha$

$\therefore \dot{x} = V\cos\alpha$

$= 30\cos\alpha$

$x = 30t\cos\alpha + c_2$ but when $t=0$

$\therefore c_2 = 0$

$\boxed{\therefore x = 30t\cos\alpha}$

b) $\ddot{y} = -g$

$= -10$

$\dot{y} = -10t + c_3$

now when $t=0$ $\dot{y} = V\sin\alpha$

$= 30\sin\alpha$

$\therefore c_3 = 30\sin\alpha$

$\dot{y} = -10t + 30\sin\alpha$

$y = -5t^2 + 30t\sin\alpha + c_4$

when $t=0$ $y=0$ ∴

$\boxed{\therefore y = -5t^2 + 30t\sin}$

$-5t^2+30t\sin\alpha$

$= +t(30\sin\alpha - 5t)$

0 or $0 = 30\sin\alpha - 5t$

$5t = 30\sin\alpha$

$t = 6\sin\alpha$

; at $t = 6\sin\alpha$

$x = (30\cos\alpha)(6\sin\alpha)$

$= 2(90\cos\alpha . \sin\alpha)$

$= 90\sin 2\alpha$ (2)

max dist, x, th~
mp can reach will
ur when $\sin 2\alpha = 1$

: the max dist is 90 m.

(1)

$= 30t\cos\alpha$

$= \frac{x}{30\cos\alpha}$ $\alpha = 69°05'$

$= -5\left(\frac{x}{30\cos\alpha}\right)^2 + 30\left(\frac{x}{30\cos\alpha}\right)\sin\alpha$

now find α when $y=0$, $x = 60$

$= -5\left(\frac{60}{30\cos\alpha}\right)^2 + 30\left(\frac{60}{30\cos\alpha}\right)\sin\alpha$

$= -5\left(\frac{4}{\cos^2\alpha}\right) + 60 \cancel{\sin}\overset{\tan}{}\alpha$

$0 = -20(\tan^2\alpha+1) + 60\tan\alpha$

$= \tan^2\alpha - 3\tan\alpha + 1$

$\tan\alpha = \frac{3 \pm \sqrt{9-4(1)(1)}}{2}$

$= \frac{3\pm\sqrt{5}}{2}$

$\alpha = 69°05', 20°54'$

OR

$x = 30t\cos\alpha$

$60 = 30t\cos\alpha$

$2 = t\cos\alpha$

$t = \frac{2}{\cos\alpha}$ $t = 6\sin\alpha$

$\frac{2}{\cos\alpha} = 6\sin\alpha$

$2 = 6\sin\alpha\cos\alpha$

$2 = 3(2\sin\alpha\cos\alpha)$

$\frac{2}{3} = \sin 2\alpha$

$2\alpha = \sin^{-1}\left(\frac{2}{3}\right)$

$2\alpha = 41.8, 138.2$

$\alpha = \frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)$

$\alpha = 20°54, 69°05''$

$V = 40$ m/s $\alpha = 0$ $g = 10$

$y = -\frac{1}{2}gt^2 + Vt\sin\alpha$

$y = -\frac{1}{2}(10)t^2 + 40t\sin 0$

$y = -5t^2$

now when $t = 0$ $y = 5$

$\therefore$

$y = -5t^2 + 5$

$x = Vt\cos\alpha$

$x = 40t\cos 0$

$x = 40t$

now when $y=0$ the pumped water reaches the ground

$0 = -5t^2 + 5$

$t = \pm 1$ $(t \geq 0)$

$\therefore t = 1$

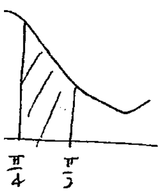
at $t = 1$ $x = 40$

$\therefore$ The water will hit the ground 40m away. It will not reach the fire.

i) Show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{8} - \frac{1}{4}$

$$\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos 2x + 1)\,dx = \frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\sin\pi + \frac{\pi}{2}\right) - \left(\frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{4}\right)\right]$$



$$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{1}{2} - \frac{\pi}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{8} - \frac{1}{4} \text{ units}^2 \quad (2)$$

$$V = \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 + \cos x)^2 \, dx \quad (1)$$

$$= \pi\int 4 + 4\cos x + \cos^2 x \, dx$$

$$= \pi\left[4x + 4\sin x + \int \cos^2 x \, dx\right]$$

$$= \pi\left[4x + 4\sin x + \int \cos(2x+1)\,dx\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \pi\left[4x + 4\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \pi\left[\frac{\pi}{8} - \frac{1}{4}\right]$$

$$= \pi\left[\left[\frac{4\pi}{2} + 4\sin\frac{\pi}{2}\right] - \left[\frac{4\pi}{4} + 4\sin\frac{\pi}{4}\right] + \pi\left[\frac{\pi}{8} - \frac{1}{4}\right]\right]$$

$=$

$$= \pi\left[\frac{9\pi}{8} + \frac{15}{4} + 2\sqrt{2}\right] u^3$$

6b) i) $v^2 = 16x - 4x^2 + 20$

$$a = \frac{d}{dx}\left[\frac{1}{2}v^2\right]$$

$$= \frac{d}{dx}\left[8x - 2x^2 + 10\right]$$

$$a = 8 - 4x$$

$$a = 4(2 - x) = -4(x - 2) \quad (2)$$

ii) centre of motion is $x = 2$

iii) when $v = 0$

$$0 = 16x - 4x^2 + 20$$
$$= -4(x^2 - 4x - 5)$$
$$= -4(x - 5)(x + 1) \quad (2)$$
$$x = 5 \text{ or } x = -1$$

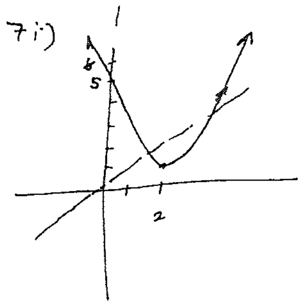
$\therefore$ The amplitude $= 3$

(iv) $n^2 = 4 \quad \therefore n = 2 \; (n > 0) \quad (1)$

v) max speed when $x = 2$ (centre of motion)

$$v^2 = 4(3 + 2 - 1)$$
$$= \pm 4 \quad (2)$$

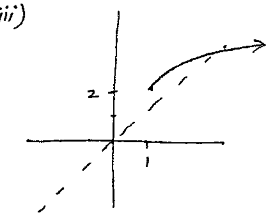
7i)



$y = (x-2)^2 + 1$

ii) $x \geq 2$ for $f^{-1}(x)$ (1)

iii)



iv) domain of inverse fn is $x \geq 1$ (1)

7a) $(1+x)^n = {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

$n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + \dots + n \cdot {}^nC_n x^{n-1}$ (1)

$x = 1$

$n \cdot 2^{n-1} = {}^nC_1 + 2\,{}^nC_2 + \dots + n\,{}^nC_n$ (1)

$$= \sum_{r=1}^{n} r \cdot {}^nC_r \quad (1)$$

v) At $x = a$, $a < 2$

$$f(a) = (a-2)^2 + 1 = A$$

$$f^{-1}(f(a)) = f^{-1}(A)$$
$$= 2 + \sqrt{A - 1}$$
$$= 2 + \sqrt{(a-2)^2 + 1 - 1}$$

$$= 2 + |(a-2)| \quad \text{since } a <$$

$$= 2 + 2 - a$$

$$= 4 - a$$

$$\therefore f^{-1}[f(a)] = 4 - a$$

vi) Since the inverse fn is the reflection of $f(x)$ about the line $y = x$, pts of int. occur when $y = x$

from $y = (x-2)^2 + 1$ & $y = x$

$$x = (x-2)^2 + 1$$
$$x = (x^2 - 4x + 4) + 1$$
$$x = x^2 - 4x + 5$$
$$0 = x^2 - 5x + 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$